A Means of Graphical Analysis for Input-Output Tables

By
Masaru Ichihashi* Hiromi Ikeda† Yoshiaki Iiguni‡

Abstract

This paper aims to present a new graphical method for use in the qualitative analysis of Input-Output (I-O) tables. Normally an effective analysis of the repercussion can be done using the Leontief inverse matrix. However, using this method, we usually can't see the way in which the interrelation among industries was attained. This is because the Leontief model is derived from the equilibrium output model. We theorize that this limits the effectiveness of Input-Output analysis since it cannot be used to analyze qualitative economic structures.

This paper analyses the economic structures of Japan under the assumption that the repercussion process is finite and continued up to the 3rd step at most. The method used in this paper aims to increase efficiency, as will be shown using graphs showing the relationships among industries, though there is some small degree of loss in theoretical strictness.This graphical analysis can be used to show the paths of the repercussion process according to the magnitude of repercussion efficiency. We believe that graphs showing the relationships among industries will be important references in making plans regarding industry policy at the state and/or national level.

1 Introduction

Input-Output analysis is usually used to estimate the efficiency of economic policy. In addition, it explains the influence of economic policy using the Leontief inverse matrix. However, there are two major questions regarding

Kochi University Review, No. 54. November 1995

*Faculty of Integrated Arts and Sciences, Hiroshima University
†Faculty of Humanities, Kochi University
‡Faculty of Humanities, Kochi University

the Leontief inverse method. The first is concerned with economic implications about the Leontief inverse method itself. The Leontief inverse method is said to show the ideal state in which the repercussion has converged, in other words, it is assumed that the repercussion process will be continued infinitely.

The second is that the process of repercussion can't be followed step by step, since the Leontief inverse is derived using the equilibrium output model. Therefore, we can't see how the results were attained.

From the point of view of economic planning, the latter question is more serious, since it means that the qualitative side of the economy can't be grasped as the result of a process of concrete repercussion.

The Leontief inverse matrix has a quantitative side which is shown as the summation of the balanced output of the sector analysed. However, a qualitative side, which shows the relationships among industries and how the output is attained, is also very important. In particular, it is indispensable when we plan which sectors or industries should grow in the regional economic policy to know the relationship among industries. Therefore, it is necessary to radically modify the classical Input-Output framework that has treated the repercussion process in such a way as to make it unclear and difficult to analyze step by step.

We'd like to present a method which expresses the I-O table graphically. There are two advantages in using this method. First, using this method, one can follow the finite steps of the repercussion process using the magnitude of efficiency. Secondly, it can clearly show the relational paths among industries as the repercussion process expands step by step. Therefore we believe that this method is effective in complementing traditional Input-Output analysis.

2 The Structural or Qualitative Analysis of I-O Table

It has been pointed out that though the Leontief inverse matrix is a optimal framework for understanding the quantitative efficiency of repercussion derived from independent demand, it is unable to show how the repercussion process affects each industry. Some methods to solve this problem have already been presented in the past.

One of these involves the triangulation of the I-O table, and was first proposed by Leontief [6]. This was a convenient method for understanding the hierarchy of industries and the structure among industries. However, triangulating from the original I-O table is complicated, and though it is an effective method for analyzing the first step in the repercussion process, it is not so useful in later steps.

On the other hand, another method that has been presented is graph theory[1] In particular, Yan and Ames [14] made wide use of this revolutionary method. They aimed to make the order matrix that is counted step order when the first transactions among sectors happened, and to compare the degree of interrelation of each region and time series data by using the index of the interrelatedness function(R). The following equation is based on that matrix.

$$R\begin{pmatrix} i_1 & \cdots & i_r \\ i_1 & \cdots & i_s \end{pmatrix} = \frac{1}{rs}\sum_{v=1}^{r}\sum_{w=1}^{s}\frac{1}{b_{i_v j_w}}. \qquad (1)$$

$1 \leq r \leq n$, $1 \leq s \leq n$, $v = 1, 2, \ldots, r$, $w = 1, 2 \ldots, s$, $b_{i_v j_w}$:elements of the order matrix

---

[1]Attempts to apply graph theory to Input-Output analysis were done comparatively early. See Rosenblatt [11], Blin and Murphy [1]. Recently, there are Olsen [9], which shows the relationship between the I-O framework and Network flow, and Holub and Schnable [3], Defourny and Thorbecke [2], Slater[2] etc. A short survey of these papers is given in Ichihashi [5].

Equation (1) expresses interrelatedness by using the mean of the reciprocal of the order matrix elements. Using this equation, the value of R increases the more sectors there are that have transactions in the early steps of repercussion process. We can see whether the degree of interrelatedness is large or small by examining the estimation of the value of R at various points in time.

However, the order matrix has the following problems.

1 . The real strength of repercussion can never be explained.
2 . Once each numerical value is put into the order matrix, the results of transaction in later steps are ignored.
3 . The paths of the repercussion process can never be expressed.

We must not neglect these problems, when planning industrial polices.

Blin and Murphy [1] presented a modification of Yan and Ames [14]s' idea that can be more simply expressed by the adjacency matrix, which has only 1 or 0 in the input coefficient matrix. Moreover, they insisted that the estimation of R is preferred to using elements of the Leontief inverse matrix because, when using the Leontief inverse matrix, it is necessary not only take into account at which step transactions have been made, but at what quantity they have been made at. This modification tried to overcome the 1st problem raised above regarding the Yan and Ames model.

However, we think that Blin and Murphy's method can never solve the 2nd and 3rd problems raised above, which must be considered when planning any form of industrial policy. In addition, they ignored Yan and Ames' contribution, that being the analysis of the qualitative interrelations among industries. Blin and Murphy put too much emphasis on the quantity of transactions and estimated the R of interrelatedness using the Leontief inverse.

We would like to suggest using the graphical method for analyzing the interrelation of industries in the following manner. The method can be used to solve the three problems mentioned above[2].

3 The Graphical Analysis for Input-Output Table

3.1 The finite repercussion process

We assume that the repercussion process will not be continued infinitly and is finite. In this subsection, we will explain the reasons for this assumption.

First, we assume a simple national economy model. It can be written as follows;

$$\boldsymbol{X} = \boldsymbol{x} + \boldsymbol{C} + \boldsymbol{I} + \boldsymbol{G} + \boldsymbol{E} - \boldsymbol{M} \tag{2}$$

$\boldsymbol{X}$ is total output, $\boldsymbol{x}$ is intermediate output, $\boldsymbol{C}$ is consumption, $\boldsymbol{I}$ is investment, $\boldsymbol{G}$ is government expenditure, $\boldsymbol{E}$ is exports, $\boldsymbol{M}$ is imports.

$\boldsymbol{C} + \boldsymbol{I} + \boldsymbol{G} + \boldsymbol{E} - \boldsymbol{M} = \boldsymbol{F}$, $\boldsymbol{x} = \boldsymbol{A}\boldsymbol{X}$ so,

$$\boldsymbol{X} = (\boldsymbol{I} - \boldsymbol{A})^{-1} \boldsymbol{F}. \tag{3}$$

This is the well-known fundamental equation used for the analysis of re-

---

[2] There were the attempts to express this using other graphs before (Slater [12], Defourny and Thorbecke [2]). However, the analysis in these studies was not complete. Our attempt may be called a general graphical method for analysis of the repercussion effects of I-O table.

By the way, Ozaki [10]'s method of "Unit Structure", another type of figure analysis, has been known as a type of structural analysis as well. This method is effective in postulating the relationships between industries in intermediate transactions. However, we think that it has two problems. The first is that this method uses the Leontief inverse matrix and the second is that we can't see the repercussion process. However, the Unit Structure method is synonymous with our following "input process graph". See Appendix B.

percussion effects in the Input-Output framework.

As mentioned before, X in equation(3) is derived from the Leontief inverse in the equilibrium state. Therefore, it isn't clear by which process it is derived. Therefore, if we want to examine in detail the repercussion process, the following equation, which is developed the Leontief inverse in series, is usually used.

$$X = (I + A + A^2 + A^3 + \cdots) F. \quad (4)$$

The series expansion on the right hand side is the formally infinite series. However, how many terms or steps are actually needed until the Leontief

Table 1 : The Percentages of The Aggregate Values in Each Steps over Inverse Elements

step number	Japan	Hiroshima pre.	Kochi	Ehime	Tokushima	Hiroshima city
1	32.9	44.3	46.1	48.0	48.8	57.6
2	62.0	79.6	78.7	79.6	82.6	89.0
3	79.7	93.3	92.7	93.0	94.9	97.4

inverse converges ?

Table 1 indicates when step matrix A converged using 1985 data on Japan, Hiroshima prefecture, Kochi prefecture, Ehime prefecture, Tokushima prefecture and Hiroshima city presented in I-O table form[3]. The numerical values in this table are the means of the percentages of the values(aggregate values) in each step[4] over each element of the Leontief inverse matrix.

It is important to note that the repercussion efficiency in the 3 rd step

[3] We used an I-O for Japan table aggregated over 32 sectors, a Hiroshima prefecture table aggregated over 38 sectors, a Hiroshima city table aggregated over 13 sectors. The other tables were aggregated over 33 sectors.

[4] The steps are each terms parenthesized in the equation (4), $A, A^2, A^3 \ldots$

explains 90 percent or more of the inverse in every table except the one for Japan. Since the self-sufficiency rate is relatively high (the rate of imports is relatively low), the values in the Japan column are lower[5]. However, if we continue using this procedure, we get values of 89.3 % in the 4 th step and 94.4 % in the 5th step. It is clear that, in the case of Japanese table, if we want to see a value for efficiency over 90 %, it is necessary to follow the path of the repercussion process to at least the 5th step[6].

It is very important for our method that the number of finite steps in the I-O table are sufficient (i.e. low step from 3 to 5) to explain most of the total repercussion[7].

3.2 Graphs of the repercussion process

As mentioned above, there are generally no problems in analyzing the continuity of repercussion efficiency in the first few steps of the I-O table. Next, we'd like to explain a method for graphical analysis of the repercussion process.

We have already mentioned the need to use eqaution (4) for examining the repercussion process step by step. However, strictly speaking, each element of A^n can never show the repercussion process. Writing the ij elements

$$\begin{array}{lll} 1\,st\ Step & a_{ij} & i,j=1,2,\ldots,n \\ 2\,st\ step & \sum_{k=1}^{n} a_{ik}a_{kj} & i,j=1,2,\ldots,n \\ 3\,rd\ Step & \sum_{s=1}^{n}\sum_{k=1}^{n} a_{ik}a_{ks}a_{sj} & i,j=1,2,\ldots,n \end{array} \qquad (5)$$

---

[5] That is, the repercussion process continues for a long time since the relationship among industries is close.

[6] However, since the values in table 1 are only the means of all industries values, each value varies with the type of industry.

[7] A similar result can be found in Ikeda [4].

of eqaution (4) step by step, it can be shown as follows;

To simplify our explanation, we use an example of a 3 sector model, matrix A^2, as follows:

$$A^2 = \begin{bmatrix} a_{11}a_{11}+a_{12}a_{21}+a_{13}a_{31} & a_{11}a_{12}+a_{12}a_{22}+a_{13}a_{32} & a_{11}a_{13}+a_{12}a_{23}+a_{13}a_{33} \\ a_{21}a_{11}+a_{22}a_{21}+a_{23}a_{31} & a_{21}a_{12}+a_{22}a_{22}+a_{23}a_{32} & a_{21}a_{13}+a_{22}a_{23}+a_{23}a_{33} \\ a_{31}a_{11}+a_{32}a_{21}+a_{33}a_{31} & a_{31}a_{12}+a_{32}a_{22}+a_{33}a_{32} & a_{31}a_{13}+a_{32}a_{23}+a_{33}a_{33} \end{bmatrix}. \quad (6)$$

Each element of the second order has been interpreted as the existence of paths and (aggregated) magnitude of repercussion. For instance, an element consisting of row 1 and column 1 is interpreted as the intermediate demand generated by the 1st sector for products from the 1st sector.

The subscript of each element on the right hand side of equation (6) makes it clear that though each ij element of shows the existence of paths from the jth sector to ith sector in the 2nd step there are three paths[8].

This explains the efficiency which an increase (assumed 1 unit per sector) in the independent final demand leads to in each sector as the result of repercussion in the 2nd step.

However, these elements can't show what sectors the repercussion passes through in the process. Equation (6) can only show the existence of paths from the jth sector to the ith sector, but not show the paths themselves. The power of input coefficient matrix can't show which paths the repercussion passes and which value of repercussion is stronger, since the values are summations of each element.

Now let's cosider the differences in the element expansion and the power of input coefficient matrix by using graphs.

The transaction between the i sector and the j sector from the 1st step to

[8] We assume that the repercussion process passes from the sector represented by the right subscript to the sector represented by the left subscript.

the 3rd step can be shown as follows;

1 *st Step*	$a_{ij} > c$	$i, j = 1, 2, \ldots, n$	$\Rightarrow$	i, j *concatenation*
2 *nd Step*	$a_{ik}a_{kj} > c$	$i, k, j = 1, 2, \ldots, n$	$\Rightarrow$	i, k, j *concatenation*
3 *rd Step*	$a_{ik}a_{ks}a_{sj} > c$	$i, k, s, j = 1, 2, \ldots, n$	$\Rightarrow$	i, k, s, j *concatenation*

(7)

Here, a_{ij} is the ij elements of the input coefficent matrix, c is the critical level value of the repercussion effect, and n is the number of sectors. Fundamentally, only equation (7) is necessary to draw our graph, which we use to call the process graph below.

Moreover, if we use a similar 3 sector matrix, $A^{[2]}$, it is as follows;

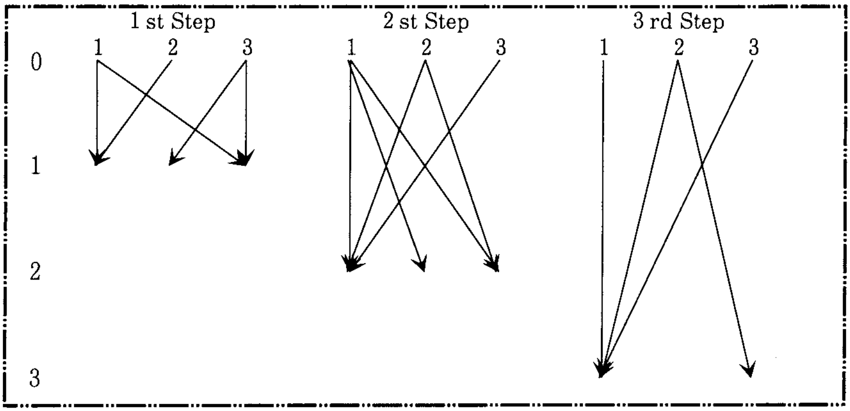
$$A^{[2]} = \begin{bmatrix} (a_{11}a_{11}, a_{11}a_{12}, a_{11}a_{13}) & (a_{12}a_{21}, a_{12}a_{22}, a_{12}a_{23}) & (a_{13}a_{31}, a_{13}a_{32}, a_{13}a_{33}) \\ (a_{21}a_{11}, a_{21}a_{12}, a_{21}a_{13}) & (a_{22}a_{21}, a_{22}a_{22}, a_{22}a_{23}) & (a_{23}a_{31}, a_{23}a_{32}, a_{23}a_{33}) \\ (a_{31}a_{11}, a_{31}a_{12}, a_{31}a_{13}) & (a_{32}a_{21}, a_{32}a_{22}, a_{32}a_{23}) & (a_{33}a_{31}, a_{33}a_{32}, a_{33}a_{33}) \end{bmatrix}.$$

Here, if we express an element of this matrix as $a_{ij}^{[n]}$, for example, $a_{12}^{[2]}$ shows that the input from the 1st sector to the 2nd sector in the 2nd step consists of three paths which are $(1 \rightarrow 2 \rightarrow 1)$,$(2 \rightarrow 2 \rightarrow 1)$ and $(3 \rightarrow 2 \rightarrow 1)$.

Now, we'd like to compare the two methods above. For example, assume the input coefficient matrix A is as follows:

$$A = \begin{bmatrix} 0.4 & 0.6 & 0 \\ 0 & 0 & 0.3 \\ 0.08 & 0 & 0.05 \end{bmatrix}. \quad (8)$$

Figure 1 : Illustration regarding the Power of the Input Coeffcient Matrix



Calculated sequentially to the 3rd step, it follows that;

$$A^2 = \begin{bmatrix} 0.16 & 0.24 & 0.18 \\ 0.024 & 0 & 0.015 \\ 0.036 & 0.048 & 0.0025 \end{bmatrix} \quad A^3 = \begin{bmatrix} 0.0784 & 0.096 & 0.081 \\ 0.0108 & 0.0144 & 0.00075 \\ 0.0146 & 0.0216 & 0.014525 \end{bmatrix}. \qquad (9)$$

The economic implication of the power of this matrix is, as we have already said, that it expresses to what degree the repercussion efficient goes when the independent final demand increase of 1 unit per sector goes to the 2nd and 3rd steps.

Figure 1 shows only the repercussion processes in which the efficiency is more than 2 %[9].

---

[9] The horizontal numbers in this figure represent individual sectors numbers of the economy. The vertical numbers correspond to the step numbers of the repercussion process. Point 0 is the initial step and the other numbers express the step numbers from one to three. The lines drawn from the upper points to the lower points show the input from the industry above going to the industry listed below. The lines drawn from the lower points to the upper points show the output of the industry in question.

It is important to note that figure 1 can't show the real paths of the repercussion process, because, in order to show them, we must first calculate the efficiency and then draw the paths one by one. By doing so, we get figure 2.

It is clear that the repercussion paths are quite different in all but the 1st repercussion in figures 1 and 2.

For example, in the case of the 2nd repercussion, the figure is drawn as if the input occurs from the 1st sector to each sector directly. Actually this repercussion consists of the paths of 1st sector(start point) → 1st sector → 1st sector, 1st sector → 3rd sector → 2nd sector and 1st sector → 1st sector → 3rd sector. Figure 1 can never show all of these paths.

The difference between the two figures isn't only in the expression of the repercussion paths. There is also the "trick" in the summing of the repercussion efficiencies.

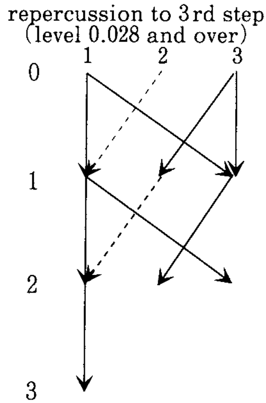
For instance, in the 3rd repercussion process in figure 1, repercussion occurrs from the 2nd sector to the 3rd sector. However, in the path graph(figure 2), no path exists from the 2nd sector to the 3rd sector in the 3rd step. The reason is that the magunitude of the repercussion was less than 0.02. However, since figure 1 shows only the aggregate values, the elements of the matrix were more than 0.02. Therefore, the path from the 2nd sector to the 3rd sector in the 3rd step is shown. This path between the 2nd and 3rd sectors in the 3rd step is a kind of the "trick" above with the aggregate values[10].

[10] In this example, we have to look closely at all 9 paths of repercussion from the 2nd sector to the 3rd sector in the 3rd step. The resulting aggregation can be quite large, though each individual value is relatively small. We called this the "trick" in the model above, but we didn't mention that the aggregate values are wrong. The framework of analysis above is based on the equilibrium model, so each element of the matrix is not dealt with here. It is assumed that all repercussions can be obtained simultaneously.

Expression (4) is based on aggregate values, so it can't show the paths that the repercussion is passing through.

Our method shows these paths only by the sequential calculations of each element.

Figure 2 : The Figure of Reperucussion Paths



The power of the coefficient matrix can't show the paths of repercussion because of the aggregation of values. Therefore, the problem of showing the process of repercussion cannot be solved by using equation (4) developed in series.

Our method can calculate all the combinations of the paths of repercussion and the magnitude of efficiency from a given coefficient matrix. And, as in figure 2, it can show the paths for the optional level of repercussion.

Each graph(from now on called "process graph") can be drawn using our method, shown below[11].

4 Analysis Based on The 1985 Japanese I-O Table

Our objective is to clearly show which points can be explained using the process graph but which cannot be adequately represented using the Leontief inverse[12 13].

---

[11] See Appendix A for an example of a process graph.

[12] We would like to express our thanks to Koji Masui, of Brain Soft Service Co. for making the process graph.

For this example, we used a Japanese I-O table based on 1985 data, but we condensed a 84 sector model to a 32 sector model as shown in table 2. We have shown the results, based on this table, in a process graph calculated up to the 3rd step[14].

Table 2 : Thei Contents of 32 Sectors

1 AFF:agricultue, forestry & fishing	19 WSS:water & sanitary services
2 MIN:mining	20 WRT:wholesale & retail trade
3 FOD:foods	21 F&I:finance & insurance
4 FTF:fabricated textiles	22 REB:real estate & bussiness services
5 P&P:pulp & paper	23 TRA:transport
6 CHE:chemical	24 C&B:communications & broadcasting
7 P&C:petroleum & coal	25 PUA:public administration
8 CSC:ceramic, stone & clay	26 E&R:education & research
9 I&S:iron & steel	27 MHS:medical & health services
10 NME:nonferrous metals	28 NOS:nonprofit organization services
11 MET:metals	29 BUS:business services
12 GME:general machiery	30 PES:personal services
13 ELE:electrical equipment	31 OFS:office supplies
14 TRE:transportation equipment	32 ANE:activities not elsewhere classified
15 PRE:precision equipment	
16 MIM:miscellaneous manufacturing	
17 CON:construction	
18 E&G:electricity & gas	

4.1 Characteristics of the process graph

Figures 3 and 4 are graphs of the multi-sectoral repercussion process[15], which represents all paths over a optional repercussion level, for a 32 sector I-O

[13] For examples of using the process graph to analyze other regions, see the example of Hiroshima city in Ichihashi[5].

[14] We need 1,108,378,624 calculations per level for a total of 15,517,300,730 pathways to calculate the values up to the 5th step (requiring 14 times in moving from the 0.03 level to the 0.002 level). It would take about 180 days for these calculations, if the average calculation time was 1,000 calculations per second. However, to calculate up to the 3rd step, we need only 1,082,368 calculations per level for a total of 15,153,152 pathways, so it would take only a few hours to make all the necessary calculations.

[15] See Appendix A concerning this kind of process graph.

table.The repercussion level of figure 3 is greater than 0.02, while the one for figure 4 is greater than 0.01.

It is clear from figure 3 that the repercussion in six sectors (fabricated textiles(4), pulp & paper(5), chemical(6), iron & steel(9), non-ferrous metals(10) and transportation equipment(14)) extends to the 3rd step. This figure shows the same sort values for the influence coeffient that are derived from the Leontief inverse. However, this method emphasizes the connection intensity among industries using lines showing these relationships rather than numbers, so is easier to understand.

Figure 3 : Multi-Sectoral Repercussion Process Graph(level:0.02)

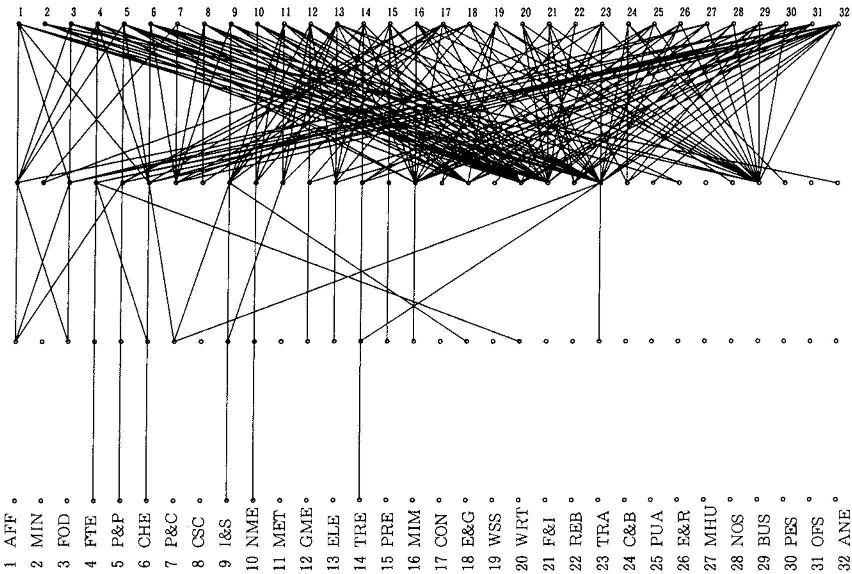


Figure 4 has a repercussion value greater than 0.01 and shows that inputs from iron & steel(9) to energy correlated industries (petroleum & coal(7) and electricity & gas(18)) occur at the 3rd step. This result cannot be shown using the Leontief inverse.

Figure 5 is an "indirect" repercussion graph which shows all the

Figure 4 : Multi-Sectoral Repercussion Process Graph(level:0.01)

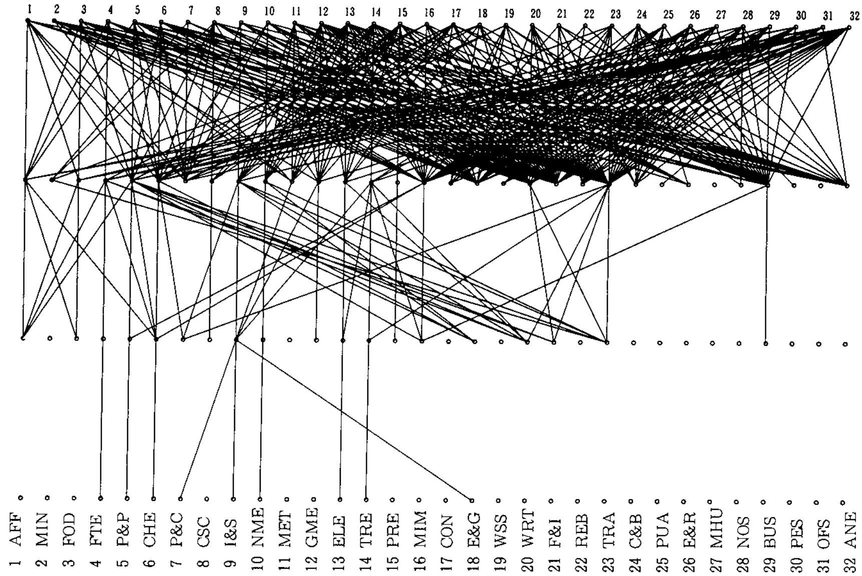
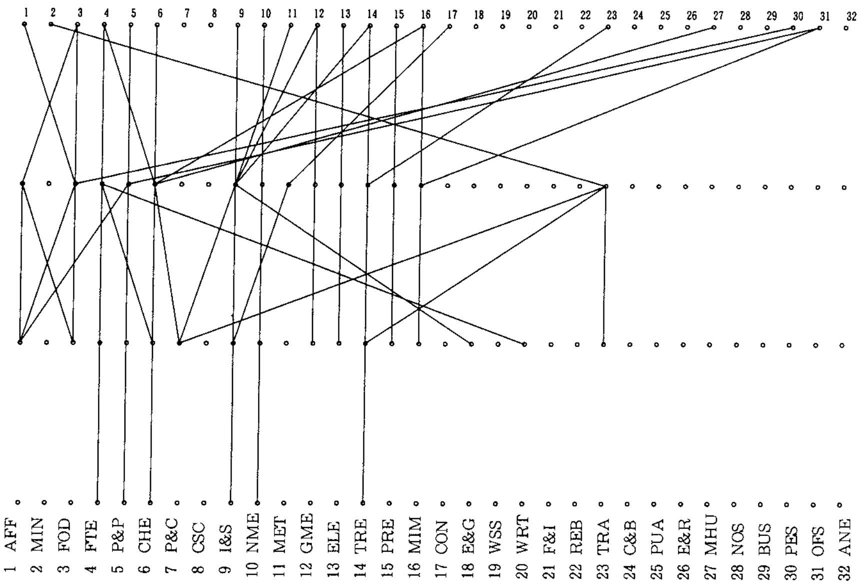


Figure 5 : Indirect Repercussion Process Graph(level:0.02)



repercussion relationships with values of 0.02 or more at the 2nd step that appeared in the multi-sectoral repercussion process graph(see Figure 3). Though service-related industries were hard to clearly analyze in the previous multi-sectoral repercussion process graph, the paths of repercussion for the transport(23), medical & health services(27), personal services(30) and office supplies(31) can be clearly seen.

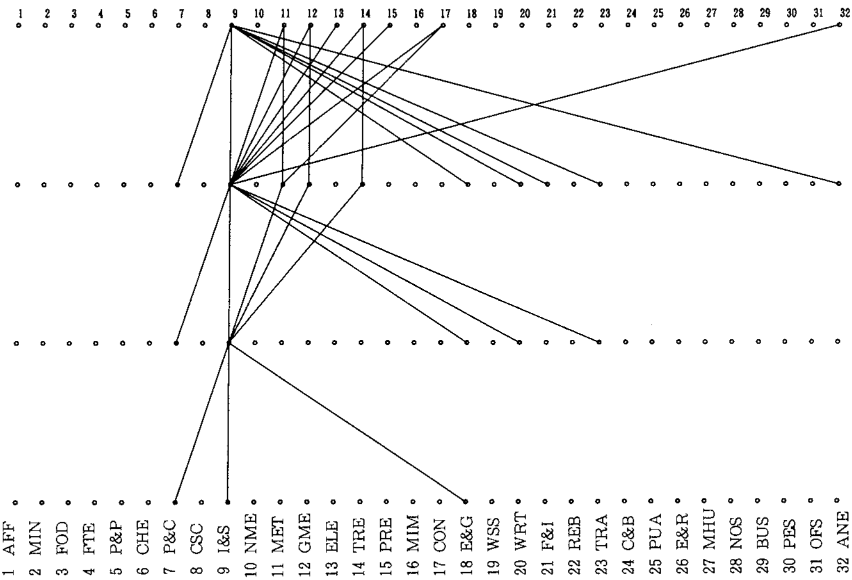
Moreover, this figure is also useful for showing the paths of the repercussion for the office supplies(31) sector, which the influence coefficient reached its maximum value as derived from the Leontief inverse.
This figure also shows ,for instance, mining(2) sector has strong influence on other sectors. Actually, the repercussion of the mining secotr(2) goes to the transporation(23) in the 1st step and to the petroleum & coal(7), transportation equipment(14) and transport(23) sectors in the 2nd step, and the repercussion of the transportation equipment sector goes to itself in the 3rd step, at a repercussion level of 0.02 or more.

4.2 Characteristics of repercussion in each industry

Next, we would like to show the repercussion paths that remain significant up to high steps in the analysis process in each industry.

We call the graph showing the input-output structure of each sector the input-output process graph. This graph brings the following two sides together. Side one shows which input paths the addition of one unit of final demand goes through. Side two shows how each output path from the initial sector is related to other sectors as indicated by the repercussion level value. For example, figure 6 shows the repercussion process in the iron & steel(9) sector has a repercussion value of 0.01 or more. This figure shows that the iron & steel sector is closely connected to the petroleum & coal, electoricity & gas, commerce and transportation sectors, and contributes output to the metals, machinery and transportation equipment sectors. This fact can be seen by analyzing data for a real-world economy. However, as seen in figure 4, this repercussion process value is larger than the repercussion value for other sectors. This point is one of the characteristics of the Japanese economy[16]. In addition, it is clear from figure 6 that the relationship of the petroleum & coal, electricity & gas, and the iron & steel industries is quite close, since the repercussion value remains relative up to the 3rd step.

Figure 6 : Input-output Process Graph of Each Sector(iron & steel, 0.01 or more)

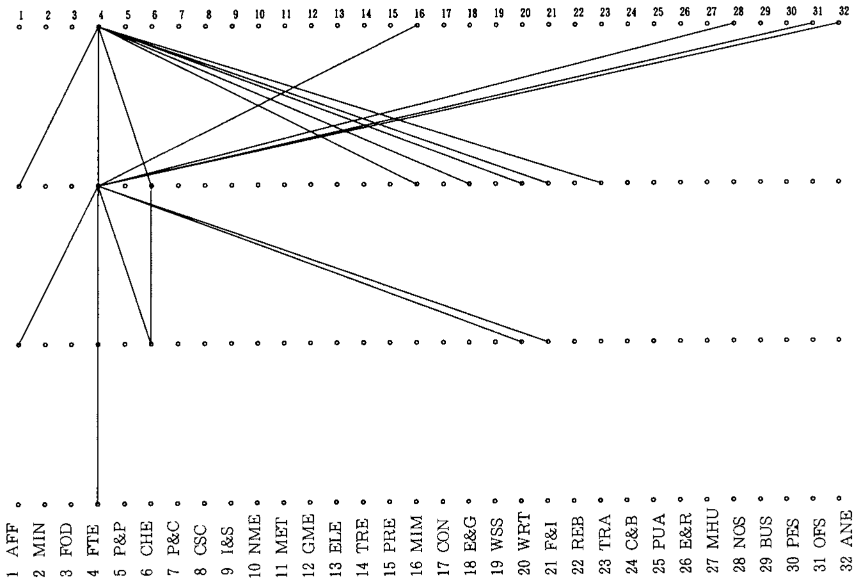


The input-output process graph for each sector is able to classify industries as one of three types. Type 1 industries are those in which the repercussions for input and output are of a similar magnitude, for example the pat-

[16] It may be surprising that "heavy industries" are still quite influential in Japan in 1985. But since this process graph is drawn by using the equation (I-M)A, which excludes the rate of import, the repercussion of other industries may extend abroad in the form of oversea's demand. To analyze this point, we must look at the original input coefficent matrix, but this is a subject for another paper. Even so, we are very interested in the fact that the repercussion from the heavy and leading industries in the 1960's like iron & steel, pulp & paper and fabricated textiles, have a relatively large effect on domestic demand.

tern for the iron & steel(9) sector in figure 6. The fabricated textiles(4), pulp & paper(5) and chemical(6) sectors also belong to the Type 1 industry group. (See figure 7,8,9. Repercussion values for type1 industries are equal to or greater than 0.01).

Figure 7 : Input-output Process Graph of Each Sector(fabricated textiles, 0.01)



Type 2 industries are those in which the repercussion paths for input are more than those of output. These types of industries are called "processing and assembly industries" and representative examples of this type of industry are the transportation equipment(14), electrical equipment(13) and office supplies(31) sectors(figures 10, 11, 12. repercussion value of 0.01 or more). In paticular, the repercussion of the office supplies sector affects the pulp & paper(5) sector in the 1st step, and agriculture, forestry & fishing, chemical, miscellaneous manufacturing, electricity & gas, wholesale & retail trade, finance & insurance and transport sectors in later steps[17].

[17] This repercussion process is difficult to analyze using the Leontief inverse and influence and response coefficient method.

Figure 8 : Input-output Process Graph of Each Sector(pulp & paper, 0.01 or more)

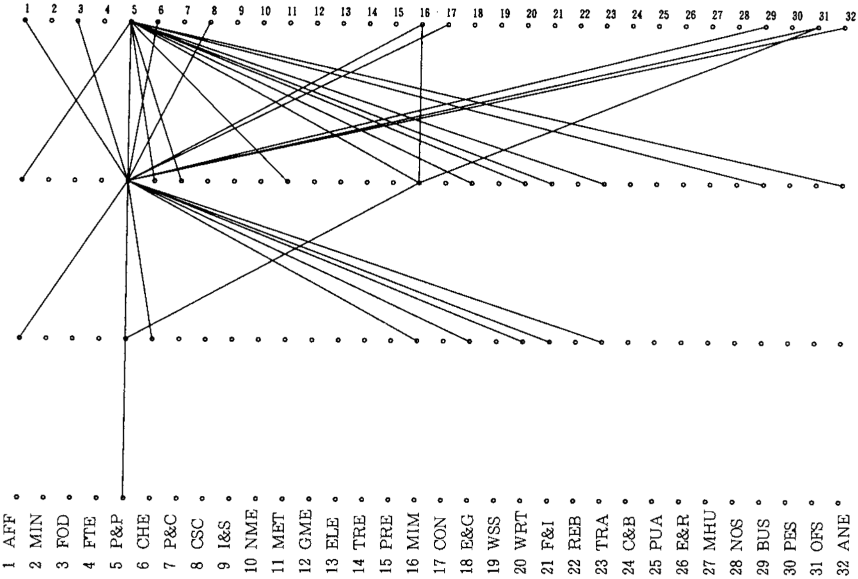


Figure 9 : Input-output Process Graph of Each Sector(chemical, 0.01 or more)

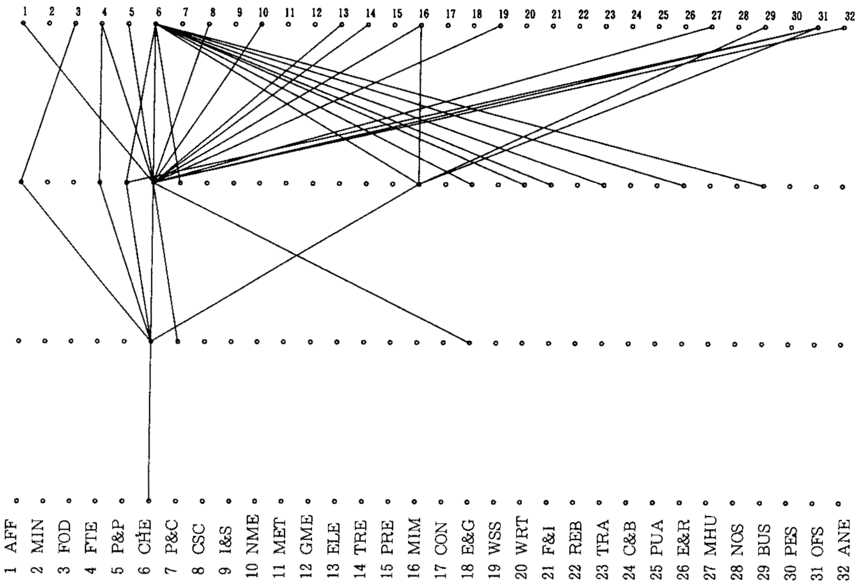


Figure 10: Input-output Process Graph of Each Sector
(transportation equipment, 0.01 or more)

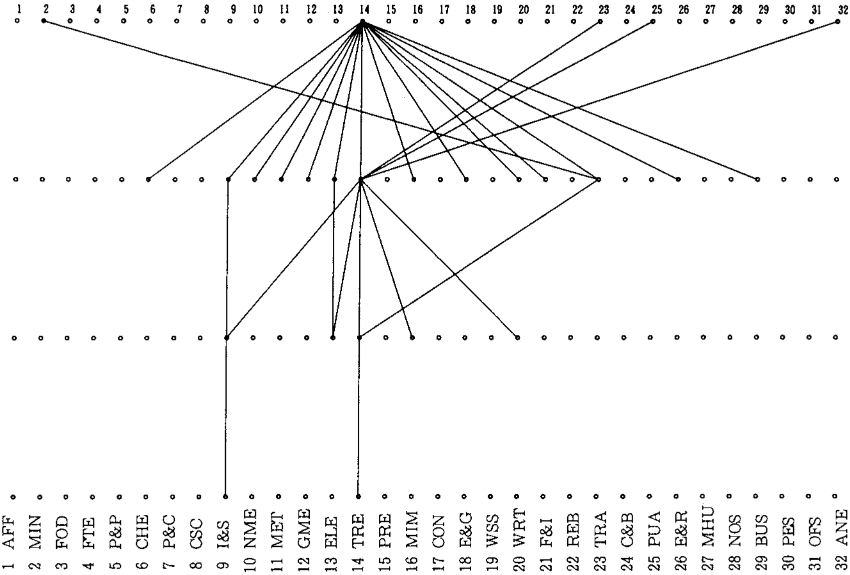


Figure 11: Input-output Process Graph of Each Sector(office supplies, 0.01 or more)

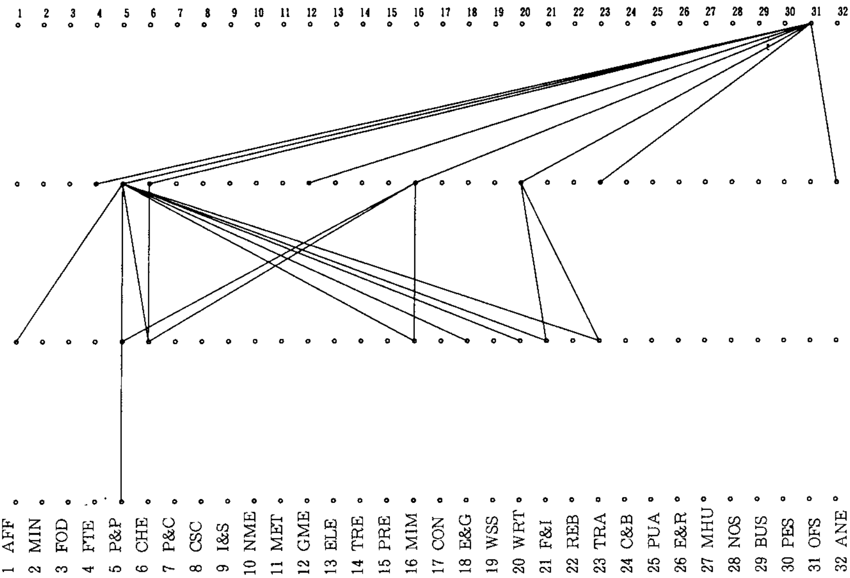
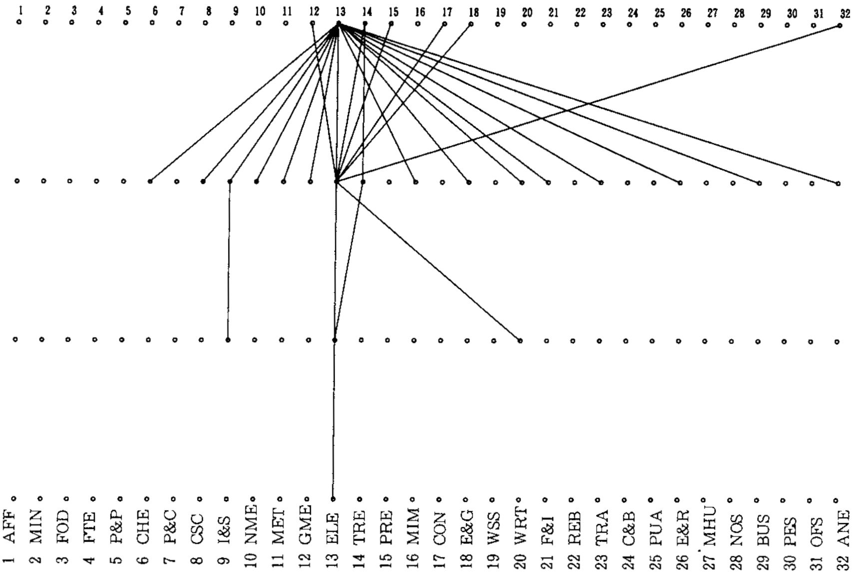


Figure 12: Input-output Process Graph of Each Sector
(electrical equipment, 0.01 or more)



The process graphs for type 3 industries show repercussion paths for output that are greater than those for input. Example of these are the petrolerum & coal(7), electricity & gas(18), transport(23) and wholesale & retail trade(20) sectors, and these are commonly called "basic industries" (see figures 13,14,15,16).

As mentioned above, since all forms of the process graph be used to analyze step-by-step the input-output relationships for each sector, it can be used quite easily to analyze the repercussion paths and the close relationships among industries.

Next, we can determine the following characteristics of sectors if we use the input process graph drawn only from the input side. Let's take a look at the input structure for the transport(23) sector. (See figure 17) Input from the transport sector to the transportation equipment sector in the 1st step goes to the 3rd step, maintaining repercussion values of 0.01 or greater.

Figure 13: Input-output Process Graph of Each Sector (petroleum & coal, 0.01 or more)

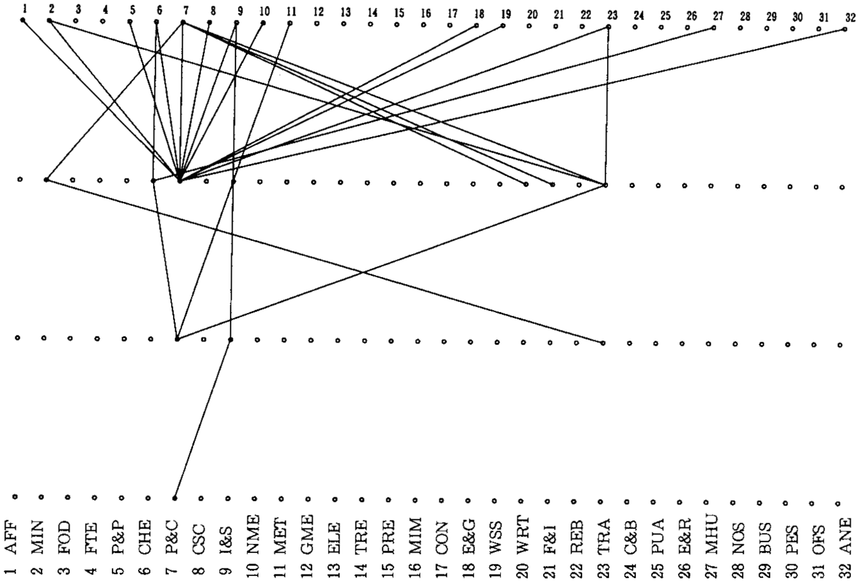


Figure 14: Input-output Process Graph of Each Sector (electricity & gas, 0.01 or more)

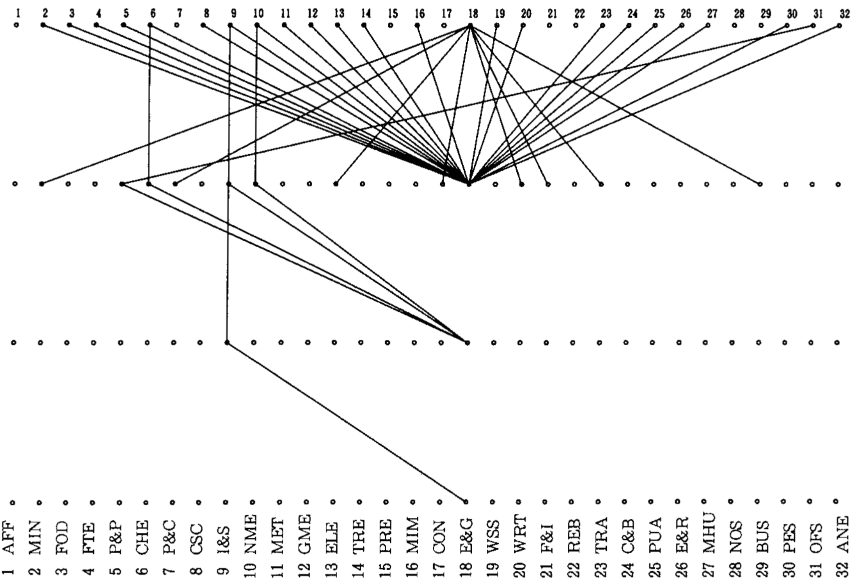


Figure 15: Input-output Process Graph of Each Sector(transport, 0.01 or more)

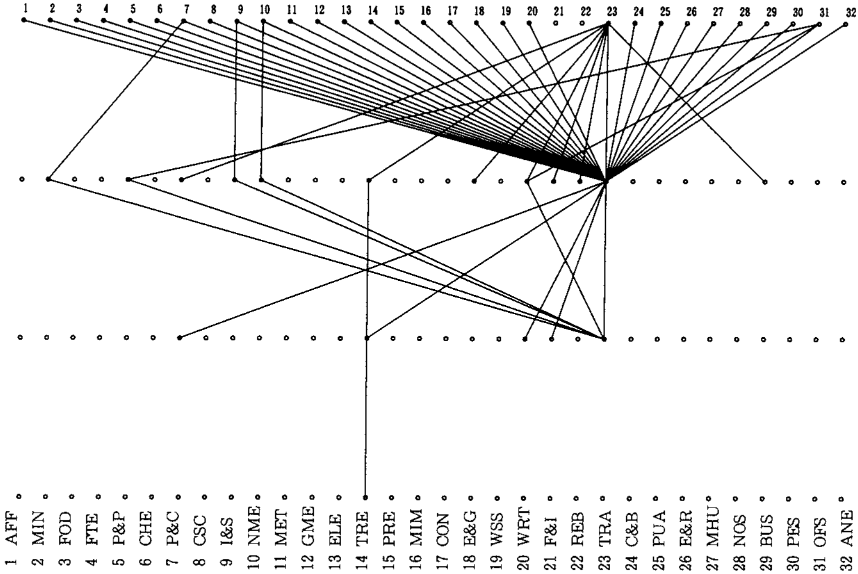


Figure 16: Input-output Process Graph of Each Sector
(wholesale & retail trade, 0.01 or more)

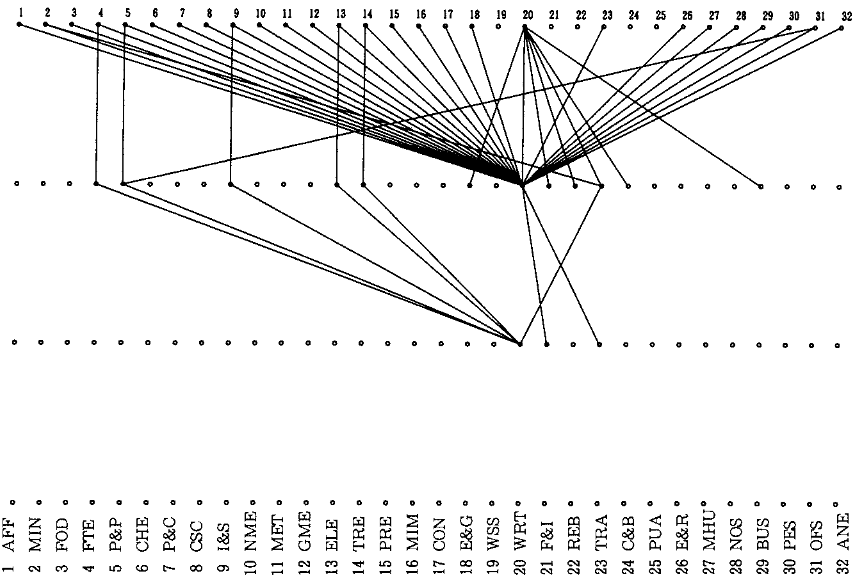


Figure 17: Input Process Graph of Each Sector(transport, 0.01 or more)

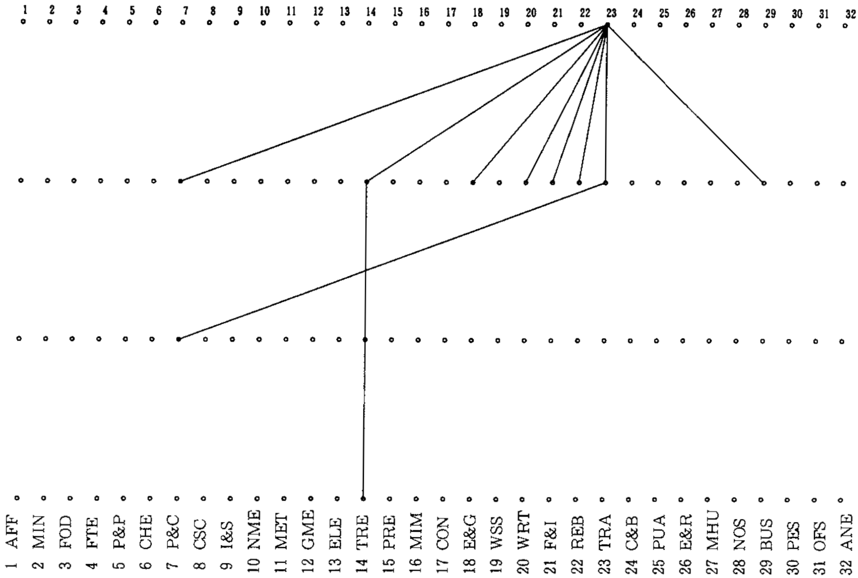


Figure 17 is different from figure 15, since figure 17 is free from the repercussion of output.

4.3 Characteristics of the maximum repercussion process graph

Figure 18 shows the maximum repercussion process graph. It is clear in this graph that the maximum repercussion of the 32 sector model extends to 16 sectors in the 3rd step.

We can see which repercussion path is the largest one in the input-output process graph for each sector above. For instance, in such sectors as the iron & steel(9), fabricated textiles(4), pulp & paper(5), chemical(6) and electrical equipment(13) sectors, the repercussion each sector has on itself is the greatest. Also in the office supplies(31) sector, the repercussion to pulp & paper(5) in the 1st step is the largest, while the one for the pulp & paper sector is largest relative to itself in the 2nd and following steps.

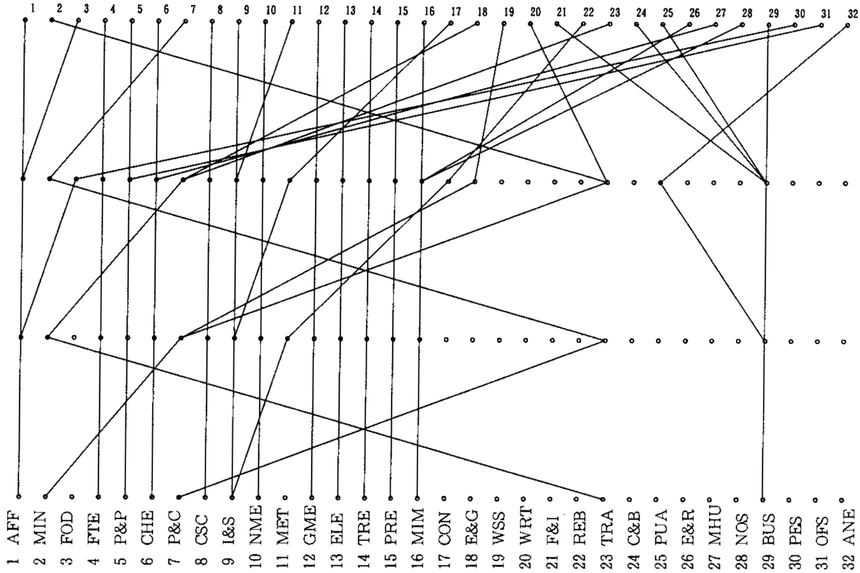
The repercussion of the wholesale & retail trade(20) sector has its greatest effects on the transport(23) sector in the 1st step, while the transport sector reaches its greatest repercussion values in relation to itself in all steps following the first.

Since the maximum repercussion process of the transport(23) sector is (transport(23)→ petroleum & coal(7) → mining(2) → transport(23)), it is clear that the maximum repercussion of the transport sector circulates back to itself in the 3rd step.

Using patterns of inter-industry relationships, one can see that the transport sector's maximum repercussion path shows the characteristics of relationships among energy related sectors in Japan. Let's examine the petroleum & coal(7) sector in the same maximum repercussion process graph. The maximum repercussion process of this sector is (petroleum & coal(7) → mining(2) → transport(23) → petroleum & coal(7)), and this is the similar circulated structure with the maximum repercussion process of the transport sector. The output paths of this sector receive input from the electricity & gas(18) and transport(23) sectors. In addition, the above sectors received maxmaum demand from the following sectors; Iron & steel(9) from petroleum & coal(7), transport(23) from the mining(2) and wholesale & retail trade(20) sectors, and electricity & gas(18) from the water & sanitary services(19) sector. Therefore, we can see that the energy related industries in Japan maintain a circular structure centered around six industries: petroleum & coal, mining, transport, wholesale & retail trade, electricity & gas and water & sanitary services[18].

[18] It is surprising that the maximum repercussion values of energy related industries in Japan are closely connected with service sectors, such as the transport, wholesale & retail trade, sanitary services sectors and, not the manufacturing sectors.

Figure18: Maximum Repercussion Process Graph



5 Concluding Remarks

We have shown a method for graphical repercussion analysis and several examples of its use.

Repercussion effect analysis using the Leontief inverse has the weak point of being unable to shown the qualitative side of the relationship among industries, though it can show the quantitive effects due to a given amount of demand. An alternative method suggested to solve this problem was the qualitative analysis method for Input-Output proposed by Yan and Ames[14].

However, this method has problems of its own. Threrefore, we suggested the above graphical method, which is based on graph theory and network flow analysis. This method is based on the assumption that the actual input coefficient matrix converges on the Leontief inverse matrix at an early level of the calculation process. The process graph can be used to show graphically the repercussion level values by using the numerical values of the input coefficient table.

Concerning the examples of the Japanese I-O table based on 1985 data, the following points were clear;

1. The repercussion for 9 industries(fabricated textiles, pulp & paper, chemical, petroleum & coal, iron & steel, non-ferrous metals, electrical equipment, transportation equipment and electricity & gas) had values at the 3rd step of 0.01 or more on the multi-sectoral repercussion process graph.
2. The indirect process graph was able to show which paths the larger repercussion values followed.
3. We looked at the sectoral repercussion paths for the iron & steel, fabricated textiles, pulp & paper, chemical, petroleum & coal, non-ferrous metals, office supplies, wholesale & retail trade and transport industries using the input-output process graph for each sector. Using this graph, we were able to classify the graph pattern as either the input-output, input or output types, and could therefore know the characteristics of the of relationships among industries.
4. The maximum repercussion process graph showed the maximum repercussion values for each of the 32 sectors analysis. In this graph, it was clear that industries relating to energy in Japan formed a circular structure composed of 6 sectors of petroleum & coal, mining, transport, electricity & gas, water & sanitary services and wholesale & retail trade.

As mentioned above, we think that some forms of the process graph are sufficiently able to analyze the qualitative sides of economic structures which traditional input-output analysis is unable to do effectively.

The problems which we confronted are as follows:

1. Comparisons of economic structures between the "closed case" and "open case" scenarios. This clarifies the economic structure in cases where a state or region can be made self-sufficient. Therefore, based on this analysis, one would be able to propose industrial policies for certain sectors with the aim of raising the rate of self-sufficiency.
2. Comparisons of interregional economies. Our analysis is able to qualitatively compare regions with similar populations and/or industrial structures. It is also useful in analyzing the qualitative differences between overcrowded and underpopulated regions (ex.Tokyo and Kochi prefecure)
3. International comparisons of similar economies. For example, Japan and Germany.
4. Comparisons of a given economy at different points in time. We can analyze the same state or region by using graphical repercussion analysis for different time periods.
5. However, the graphical repercussion analysis doesn't have the index necessary for measuring the similarity between the structures of various types of industry. If some index concerning these could be made, our method would be more effective.

References

[1] Blin, J. M. and Murphy, F., "Notes and Comments On Measuring Economic Interrelatedness," *Review of Economic Studies*, Vol. 61, 1974, pp. 437–440.

[2] Defourny, J. and Thorbecke, E., "Structural Path Analysis and

Multiplier Decomposition Within a Social Accounting Matrix Framework, " *The Economic Journal*, 94, March, 1984, pp. 111−136.

[3] Holub, H. W. and Schnabl, H., "Qualitative input-output analysis and structural information," *Economic Modelling*, Jan., 1985, pp. 67−74.

[4] Ikeda, H., "Structural analysis for regional industrial policy," (in Japanese), *Kochi University Review of Social Science,* No.49, Mar., 1994.

[5] Ichihashi, M., "Graph theory and process graphs" (in Japanese), *Kinoshita, Ohnishi and others ed., Keizai Toukeigaku no Gendaika,* Kouyou syobou, 1995.

[6] Leontief, W., "The Structure of Development," *Input-Output Economics 2nd ed.*, chapter 8, Oxford Univ. Press 1986.

[7] Niida, H., *Introduction to Input-Output Analysis* (in Japanease), Toyo keizai shinpousya, 1980.

[8] Nozaki, A., *Risan kei no sugaku* (in Japanease), Kindai kagakusya, 1980.

[9] Olsen, J. A., "Input-output models, directed graphs and flows in networks," *Economic Modelling,* Oct., 1992, pp. 365−384.

[10] Ozaki, I., "The Structure of Economic Development (Ⅲ) −A Statistical Determination of Economic Fundamental Structure − ," (in Japanese), *Mita Journal of Economics*, Vol. 73, No. 5, Oct., 1980.

[11] Rosenblatt, D., "On Linear Models and the Graph of Minkowski-Leontief Matrices," *Econometrica,* Vol. 25, No. 2, April, 1957, pp. 325 − 338.

[12] Slater, P. B., "The Determination of Groups of Functionally Integrated Industries in the United States Using a 1967 Interindustry Flow Table," *Empirical Economics,* Vol. 2, 1977, pp. 1−9.

[13] Wilson, R. J., *Introduction to Graph Theory,* 3rd edition, Longman Group Limited, London, England, 1985 .

[14] Yan, C. and Ames, E., "Economic Interrelatedness," *Review of Economic Studies,* Vol. 32, 1965, pp. 299−310.

[15] Yokokura, H., *the Introduction to Input-Output Analysis* (in Japanease), Madosya, 1990.

Appendix

A The Various Types of Process Graphs

Some kinds of repercussion process graphs are as follows.

1 . Multi-sectoral repercussion process graph

This graph is the most basic one used in our method of analysis. This can be drawn using equation (7).The multi-sectoral repercussion process graph shows the repercussion efficiency process at the optional level step by step. We can see not only the values of the repercussion, but also the quality as it relates to the economic structure shown in the graph.

2 . Indirect repercussion process graph

The indirect repercussion process graph shows only the repercussion remaining in the 2nd and later steps. In other words,

$$\begin{array}{llll} 2\,nd\ Step & a_{ik}a_{kj} > c & i,k,j = 1,2,\ldots,n & \Rightarrow\ i,k,j\ concatenation \\ 3\,rd\ Step & a_{ik}a_{ks}a_{sj} > c & i,k,s,j = 1,2,\ldots,n & \Rightarrow\ i,k,s,j\ concatenation \end{array} \tag{10}$$

Since the direct repercussion is clear from the input coefficient matrix, this graph, which shows repercussion maintained over multiple steps, is actually very useful. Also, this graph is much simpler than the multi-sectoral repercussion process graph.

3 . Input-output process graph for each sector

The multi-sectoral and indirect repercussion process graph are used to follow the repercussion process of all sectors simultaneously. However, the input-output process graph can also be used to show only the repercussion values of the sectors we want to see. This graph can illustrate the input paths from, and the output paths to, a sector at the same time. The equation of the graph for the ith sector is as follows;

$$\begin{array}{llll} 1\,st\ Step & a_{ij} > c & j=1,2,\ldots,n & \Rightarrow\ i,j\ concatenation \\ 2\,nd\ Step & a_{jk}a_{ki} > c & j,k=1,2,\ldots,n & \Rightarrow\ i,k,j\ concatenation \\ 3\,rd\ Step & a_{jk}a_{ks}a_{si} > c & j,k,s=1,2,\ldots,n & \Rightarrow\ j,k,s\ concatenation \end{array} \quad (11)$$

and

$$\begin{array}{llll} 1\,st\ Step & a_{ij} > c & i,j=1,2,\ldots,n & \Rightarrow\ i,j\ concatenation \\ 2\,nd\ Step & a_{ik}a_{kj} > c & k,j=1,2,\ldots,n & \Rightarrow\ i,k,j\ concatenation \\ 3\,rd\ Step & a_{ik}a_{ks}a_{sj} > c & k,s,j=1,2,\ldots,n & \Rightarrow\ i,k,s,j\ concatenation \end{array} \quad (12)$$

Moreover, this graph can be used to show only the input process graph for each sector as well. This input process graph illustrates what input occurs and how the repercussion expands when there is a unit of demand added to the sector in question. Therefore, it shows the input structure for each sector[19].

4 . Process graph for each step

This graph shows the step paths of the multi-sectoral repercussion process graph. This is used to pick out the step we wish to analyze. The equation is as follows;

$$lth\ Step \quad a_{ik}a_{ks}\ldots a_{mj} > c \quad i,k,s,\ldots,m,j=1,2,\ldots,n$$
$$\Rightarrow\ i,k,s,\ldots,m,j\ \ concatenation \quad l \geq 1\,. \quad (13)$$

[19]For example, see figure 17. Also, this input process graph is similar to Ozaki's Unit Structure(or Unit System)[10]. See Appendix B regarding this point.

The value of l must be equal to the number of terms of multiplication in equation (13). This graph is drawn calculating each step as $l, l-1, \ldots, 2, 1$ of equation (13).

5. Maximum repercussion process graph

The four graphs above are diagrams used to analyze the process of repercussion efficiency at each level. However, we can show the maximum repercussion paths for each sector, as well. This graph is useful when we need to determine in which sectors the largest efficiency values occurs for each sector analyzed.

It is helpful to use the special adjacency matrix $\boldsymbol{W}$ to understand the principles of the maximum repercussion process graph.

$$\boldsymbol{W} = [w_{ij}] \tag{14}$$

$$w_{ij} = \begin{cases} 1\ (max\ a_{ij}) \\ 0\ (others) \end{cases} \quad i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n. \tag{15}$$

$$l th\ Step \quad w_{ik}w_{ks} \ldots w_{mj} = 1 \quad i, k, s, \ldots, m = 1, 2, \ldots, n$$
$$\Rightarrow \quad i, k, s, \ldots, m, j \quad concatenation \quad l \geq 1, j = 1, 2, \ldots, n. \tag{16}$$

B Unit Structure and Input Process Graph

As mentioned in footnote 2, Ozaki[10]'s Unit Structure is actually quite similar to our input process graph.

Ozaki's Unit Structure is written as;

$$\boldsymbol{U}_j = \boldsymbol{A}\boldsymbol{B}. \tag{17}$$

$\boldsymbol{A}$ is the input coefficient table, $\boldsymbol{B}$ is the diagonal matrix composed of elements in the columns of the Leontief inverse matrix.

Now, assuming the final demand of $\boldsymbol{F}_j$ is the column vector that yields only 1 unit of the jth sector (*i.e.* output of other sectors are 0), the total output, X_j, is;

$$X_j = (I - A)^{-1} F_j. \tag{18}$$

This equation is derived from the equation,

$$X_j = A X_j + F_j. \tag{19}$$

Now, substituting (18) for right hand side of (19) yields,

$$X_j = A(I - A)^{-1} F_j + F_j. \tag{20}$$

The first term $A(I - A)^{-1} F_j$ on the right hand side of the equation is the part that corresponds to the Unit Structure above.However, the two are not completely identical.

The Unit Structure is none other than the matrix($n \times n$) expressed by sectors, and not the aggregate value showing how the influence (indirect repercussion) input in the optional sector(here it is the jth sector) influenced each sector. This takes apart and expresses by each sector the aggregate value of influence from the input of jth sector to all sectors as indicated by $A(I - A)^{-1} F_j$ (equation (20)). In this way, the Unit Structure and the first term of (20) can become quite similar.

Moreover, this equation(20) can be developed as follows:

$$X_j = A(I + A + A^2 + \cdots) F_j + F_j. \tag{20}$$

The first term on the right hand side shows the repercussion process when the jth sector receives one unit of additional input. This is also used in our input process graph. As mentioned above, our input process graph is the graph of repercussion by sector, not the aggregate value. Therefore, it may be said that the repercussion process graph can be said to be an improved version of Ozaki's Unit Structure.

However, our method can be used to illustrate not only the input process graph, but other kinds of graphs as well. Therefore, our method is a more general method of structure analysis that can include the Unit Structure model as a sort of special case.