Shear viscosity of the quark matter

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Abstract. We discuss shear viscosity of the quark matter by using Kubo formula and the Nambu-Jona-Lasinio model. It is shown that the leading term of 1/N expansion for the shear viscosity is equivalent to the quasi-particle random phase (RPA) approximation. Using this approximation, we obtain a formula that the shear viscosity is expressed by the quadratic form of the quark spectral function in the chiral symmetric phase. If we assume the Breit-Wigner type for the spectral function, the shear viscosity is evaluated as a function of the imaginary part of the quark self energy, which is related to the mean free path of the quark. It is pointed out that the quark matter seems to be a perfect liquid for the mean free path $< 2\,\mathrm{fm}$.

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1. Introduction

The existence of the quark gluon plasma (QGP), which is predicted by Quantum chromodynamics, has not been discovered in Nature. In order to produce such a new state of matter, the experimental investigation started at the Relativistic Heavy Ion Collider (RHIC). The data at RHIC, however, seems to reveal some unexpected properties of the high density matter produced in the experiment[1]-[4]. Namely it could be explained by a fluid model with small viscosity; it is almost perfect fluid. This fact encourages many researchers to calculate the transport coefficients of the quark matter[5]-[20]. However their calculations are so complex that the numerical results are not settled completely. On the other hand, many studies by the lattice QCD have been in progress[21]-[22] Recently an approach from a very quite different field, black hole physics, has generated a great deal of interest. This theory is based on the AdS/CFT correspondence and predicts the lower bound of the shear viscosity of the quark matter[23]-[25].

It is the purpose of this paper to calculate the shear viscosity of the quark matter using Kubo formula[26]-[29]. It is known that a free quark gas has *infinite* shear viscosity. It is the strong interaction between quarks that lowers this infinite viscosity. We are interested in the physical origin of the finite (small) viscosity. Hence we take up only the quark sector and use the Nambu-Jona-Lasinio (NJL) model in this paper. In order to calculate the correlation function in the Kubo formula, we use the quasi-particle random phase approximation (RPA). It is shown that this approximation is equivalent to the leading term in the large N expansion, which is popular in hadron physics. Using this approximation, we obtain the shear viscosity expressed by the quadratic form of the quark spectral function. In this paper, we assume the Breit-Wigner type for the spectral function instead of calculating it. Since the imaginary part of the quark self energy is related to the mean free path of the quark, the shear viscosity is evaluated as a function of the mean free path. Moreover we discuss the condition that the quark matter can be regarded as a perfect liquid by applying the concept of the Reynolds number to the quark matter.

In the next section, the Kubo formula for the shear viscosity is reviewed. Then it is calculated with the use of the quasi-particle RPA in §3. Finally assuming the Breit-Wigner type for the spectral function, the shear viscosity is evaluated and several comments are given.

2. Kubo Formula for Shear viscosity

According to the Kubo formula, the shear viscosity $\eta(\omega)$ at temperature T is given by

$$\eta(\omega) = \frac{1}{T}\int_0^\infty \mathrm{d}t \mathrm{e}^{i\omega t} \int \mathrm{d}\mathbf{r}(J_{xy}(\mathbf{r},t), J_{xy}(0,0)), \tag{1}$$

where J_{xy} is the x, y component of the energy-momentum tensor of the quark matter. The correlation function in the right-hand side is defined by

$$(A, B) \equiv \beta^{-1} \int_0^\beta \mathrm{d}\lambda \langle \mathrm{e}^{\lambda H} A \mathrm{e}^{-\lambda H} B \rangle, \tag{2}$$

where A and B are operators of any physical quantity and H denotes our Hamiltonian. The bracket, $\langle A \rangle = \mathrm{Tr}(A\mathrm{e}^{-\beta H})/\mathrm{Tr}\mathrm{e}^{-\beta H}$, means the thermal average at temperature T ($\beta \equiv 1/T$). Using partial integration in the right-hand side of Eq.(1), the viscosity can be transformed into

$$\eta(\omega) = \frac{i}{\omega}[\Pi^{\mathrm{R}}(\omega) - \Pi^{\mathrm{R}}(0)]. \tag{3}$$

Here $\Pi^R(\omega)$ is a retarded Green's function defined by

$$\Pi^R(\omega) = -i \int_0^\infty \mathrm{d}t \mathrm{e}^{i\omega t} \int \mathrm{d}\mathbf{r} \langle [J_{xy}(\mathbf{r}, t), J_{xy}(0, 0)] \rangle, \tag{4}$$

where [,] in the integrand denotes the commutation relation. Noting that $(\Pi^R(\omega))^* = \Pi^R(-\omega)$, the (static) viscosity is reduced to

$$\eta \equiv \eta(\omega = 0) = -\frac{d}{d\omega} \mathrm{Im}\Pi^R(\omega) \Big|_{\omega=+0}. \tag{5}$$

In order to calculate the above $\Pi^R(\omega)$, it is convenient to transform into the imaginary time (Matsubara) formalism. We introduce the following correlation function,

$$\Pi(i\omega_n) = -\int_0^\beta \mathrm{d}\tau \mathrm{e}^{-i\omega_n \tau} \int \mathrm{d}\mathbf{r} \langle T_\tau (J_{xy}(\mathbf{r}, \tau) J_{xy}(0, 0)) \rangle, \tag{6}$$

where the Matsubara frequency is represented by $\omega_n = 2\pi n T$ and T_τ means the (imaginary) time ordering operator. As is well known, the retarded Green's function $\Pi^R(\omega)$ is obtained by the analytic continuation: $\Pi^R(\omega) = \Pi(i\omega_n)|_{i\omega_n = \omega + i\delta}$.

Since we are interested in the quark sector as mentioned in §1, we take the Nambu-Jona-Lasinio (NJL) model in this paper[30]. The Lagrangian density is given by

$$\mathcal{L} = \bar{\psi}(i\gamma \cdot \partial - m)\psi + g[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \tau \psi)^2]. \tag{7}$$

Here ψ is the field operator for quarks and the quark mass is assumed to be zero ($m = 0$). Then the canonical energy-momentum tensor is read as

$$J_{xy} = \frac{i}{2}[\bar{\psi}\gamma^2 \partial^1 \psi - \partial^1 \bar{\psi}\gamma^2 \psi]. \tag{8}$$

If this expression is substituted into Eq.(6), the correlation function Π is calculated according to the Wick theorem (Feynman diagram method). Detailed calculation will be given in the next section.

3. Random Phase Approximation and 1/N Expansion

In this section, we will calculate the correlation function (6) by using Feynman diagram method. The interaction of NJL model is represented by the Feynman diagram Fig.1 where Γ is 1 (unit matrix) or $i\gamma_5\tau$. Note that the four Fermi operators are composed of two pairs which are connected by a broken line.

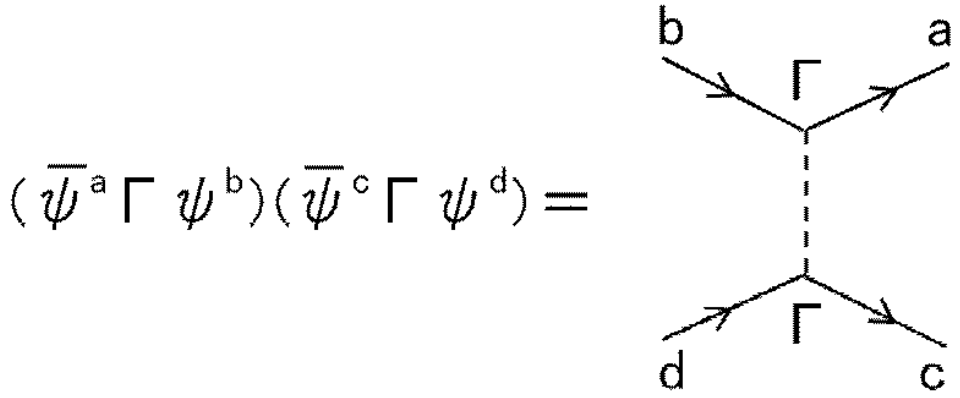

Figure 1. The diagram for the interaction with $\Gamma = 1$ or $i\gamma_5\tau$

Now we take the Fermi liquid theory and the random phase approximation (quasi-particle RPA). Then the correlation function is approximated by the ring diagrams as shown in Fig.2. All the propagators must be the dressed ones because the free propagator gives rise to the infinity in the first term of Fig.2 as shown later. Therefore we must take the full propagator for each quark line in this approximation. In fact the quark has strong correlation and the imaginary part of the self energy even in the chiral symmetric phase[31]. A similar calculation was done in a relativistic scalar field theory in the Ref.(30) where the shear viscosity was calculated by taking into account chain diagrams.

At first we show that the RPA is derived by the $1/N$ expansion popular in QCD ($N = N_c$)[33]-[34]. Let a diagram has n vertices and n_l quark loops. Let us consider the N-dependence of this diagram. Each quark loop gives rise to N from the trace of color variable. The coupling constant of the NJL model is order of $1/N$ because the quark-gluon coupling constant is order of $1/\sqrt{N}$ in the $1/N$ expansion. Consequently this diagram is order of N^{n_l-n}. Therefore the leading diagrams are those with the maximum number of the loop for the fixed n. It is proved that $n_l \leq n+1$.

proof Let us consider any diagram with n vertices. If the number of the quark propagators is n_i, we have the relation, $2n_i = 4 + 4n$. Each propagator belongs to only one loop necessarily according to the Wick theorem. Since each loop contains two or more propagators, the maximum number of the loop is realized that all the loops are composed of two propagators: $n_l = n_i/2 = n+1$. The loop composed of one propagator

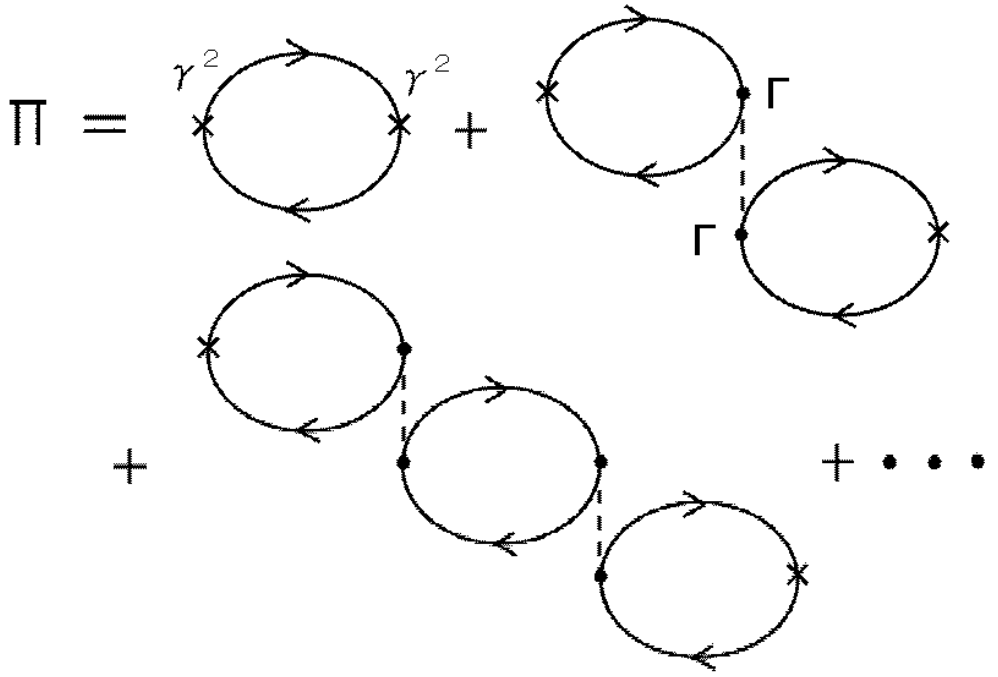

Figure 2. The ring diagrams for the correlation function Π.

(tadpole) is included in the dressed propagator. Thus it is evident that such diagrams correspond to the ring ones of Fig.2 and their magnitude is $O(N^1)$.

For example, let us consider the first order diagrams with respect to the interaction shown in Fig.3. One has two loops (Hartree term) and the other has only one loop (Fock term) composed of four propagators. The former (a) is $O(N^1)$ because it has one vertex and two quark loops. The latter (b) with one vertex and one loop is $O(1)$. Consequently the latter term should be neglected. In the same way the third diagram in Fig.2 is order of N and so on.

Now the correlation function is written as

$$\begin{aligned}\Pi(i\omega_n) = &\frac{1}{\beta}\sum_l \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} {p_x}^2 \mathrm{Tr}[\gamma^2 G(\mathbf{p},\omega_l+\omega_n)\gamma^2 G(\mathbf{p},\omega_l)] \\ &+ \frac{2g}{\beta}\sum_{l,m}\int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3} p_x q_x \mathrm{Tr}[\gamma^2 G(\mathbf{p},\omega_l+\omega_n)\Gamma G(\mathbf{p},\omega_l)] \\ &\times \mathrm{Tr}[\Gamma G(\mathbf{q},\omega_m-\omega_n)\gamma^2 G(\mathbf{q},\omega_m)]\cdots, \end{aligned} \tag{9}$$

where $G(\mathbf{p}, i\omega_n)$ is the dressed quark propagator. It should be noted that the dressed propagator is proportional to the linear combination of γ^μ in the chiral limit[36]. This leads to $\mathrm{Tr}[\gamma^2 G\Gamma G] = 0$ in the second term because the trace has an odd product of γ matrices ($\Gamma = 1$ or $i\gamma_5\tau$). Thus the second term vanishes. Similarly the third term contains the same loop with odd product of γ matrices so that it vanishes. The same calculation is realized in all the higher order terms in Fig.2. Consequently the correlation function Π is reduced to only the first term in our present case.

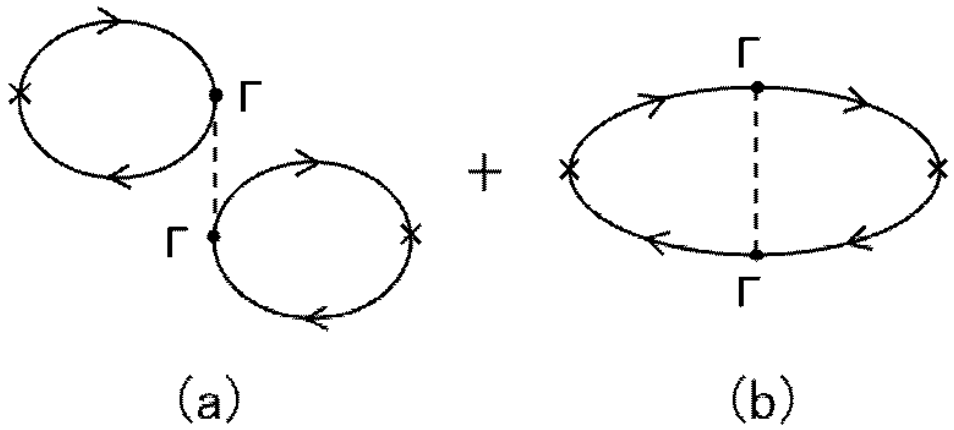

Figure 3. The first order diagrams: (a) Hartree term and (b) Fock term.

In order to calculate the correlation function, let us follow the procedure taken for the calculation of the electrical conductivity in Ref[35]. We consider the spectral representation for the dressed propagator, which is written as

$$G_{\alpha\beta}(\mathbf{p}, i\omega_l) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi} \frac{\rho_{\alpha\beta}(\mathbf{p}, \varepsilon)}{i\omega_l - \varepsilon}. \tag{10}$$

We substitute this expression into the correlation function (Eq.(8)) and the summation over Matsubara frequency is replaced by the contour integral:

$$\begin{aligned} S &\equiv T \sum_l \mathrm{Tr}[\gamma^2 G(\mathbf{p}, i\omega_l + i\omega_n)\gamma^2 G(\mathbf{p}, i\omega_l)] \\ &= -\int_C \frac{\mathrm{d}z}{2\pi i} n(z) \mathrm{Tr}[G(\mathbf{p}, z)\gamma^2 G(\mathbf{p}, z + i\omega_n)\gamma^2], \end{aligned} \tag{11}$$

where $n(z) = (1 + e^{\beta z})^{-1}$ is the Fermi distribution function. The contour C is divided into three pieces as shown in Fig.4, because the integrand has branch cuts on the two lines $z = \varepsilon$ and $z = \varepsilon - i\omega_n$ where ε is real (Note that poles of $n(z)$ do not lie on the two branch cuts).

Since the integral along the large circle vanishes, it is rewritten as

$$\begin{aligned} S = -\int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi i} n(\varepsilon) \mathrm{Tr}[&G(\varepsilon + i\delta)\gamma^2 G(\varepsilon + i\omega_n)\gamma^2 - G(\varepsilon - i\delta)\gamma^2 G(\varepsilon + i\omega_n)\gamma^2 \\ &+ G(\varepsilon - i\omega_n)\gamma^2 G(\varepsilon + i\delta)\gamma^2 - G(\varepsilon - i\omega_n)\gamma^2 G(\varepsilon - i\delta)\gamma^2], \end{aligned}$$

where δ is an infinitesimal positive number introduced in order to avoid the branch cuts. Here the argument $\mathbf{p}$ is abbreviated for convenience. Noting the relation,

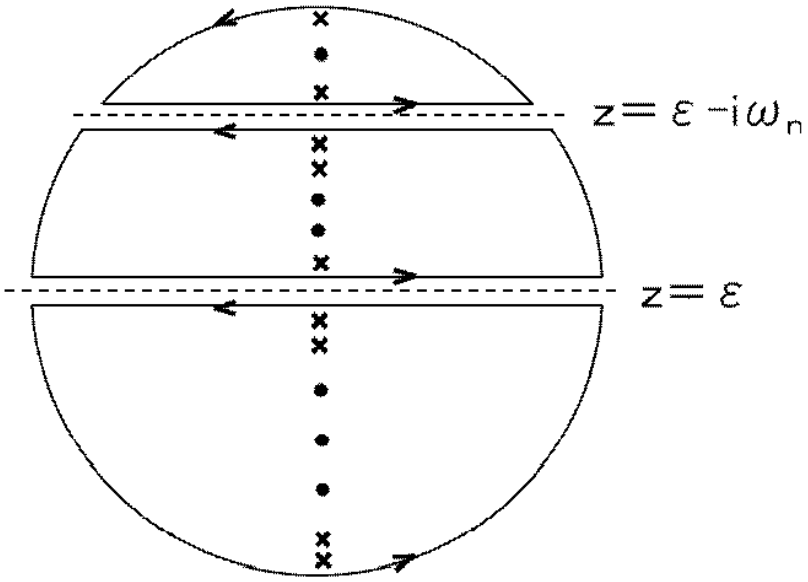

Figure 4. The contour for the calculating the integral of z in Eq.(11).

$G(\varepsilon + i\delta) - G(\varepsilon - i\delta) = -i\rho(\varepsilon)$, the above equation is expressed as follows:

$$S = \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi} n(\varepsilon) \mathrm{Tr}[(G(\varepsilon + i\omega_n) + G(\varepsilon - i\omega_n))\gamma^2 \rho(\varepsilon)\gamma^2]. \tag{12}$$

Substituting the spectral representation (10) again, we get

$$S = \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon'}{2\pi} n(\varepsilon) \left(\frac{1}{\varepsilon + i\omega_n - \varepsilon'} + \frac{1}{\varepsilon - i\omega_n - \varepsilon'} \right) \mathrm{Tr}[\rho(\varepsilon')\gamma^2 \rho(\varepsilon)\gamma^2]. \tag{13}$$

By the analytic continuation $i\omega_n \longrightarrow \omega + i\delta$, the imaginary part of the above equation becomes

$$\mathrm{Im}S = \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi} \frac{1}{2} (n(\varepsilon + \omega) - n(\varepsilon)) \mathrm{Tr}[\rho(\varepsilon + \omega)\gamma^2 \rho(\varepsilon)\gamma^2], \tag{14}$$

because $\mathrm{Tr}[\rho(\varepsilon')\gamma^2\rho(\varepsilon)\gamma^2]$ is real. As a result we obtain an expression for the shear viscosity which is related to the quark spectral function,

$$\eta = -\frac{1}{2} \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi} \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} {p_x}^2 \frac{\partial n}{\partial \varepsilon} \mathrm{Tr}[\rho(\mathbf{p}, \varepsilon)\gamma^2 \rho(\mathbf{p}, \varepsilon)\gamma^2]. \tag{15}$$

This equation means that the calculation of the shear viscosity is reduced to that of the spectral function. The spectral function has a peak near $\varepsilon = \pm p - \mu$ from Eq.(19). Then the factor $\partial n/\partial\varepsilon$ has also a peak near the Fermi surface and decreases exponentially with $|p \mp \mu|$, which leads to the convergence of the above integral. On the other hand, the integral with respect to the momentum has a cutoff in the NJL model.

4. Numerical Results

Now we are in a position to discuss the quark spectral function when the chiral symmetry is restored. According to Weldon[36], the general forms of the self-energy for the massless quark is expressed by

$$\Sigma(p) = \Sigma_+\gamma^0\Lambda_+ + \Sigma_-\gamma^0\Lambda_-, \tag{16}$$

using the projection operator $\Lambda_\pm = (1 \pm \hat{\mathbf{p}}\mathbf{r})$ and $p_0 \equiv \varepsilon + \mu$ (μ: the chemical potential). Then the retarded (advanced) Green's function are given by

$$\begin{aligned} G^R(p) &= \frac{\gamma^0\Lambda_+}{p_+^0 - p - \Sigma_-} + \frac{\gamma^0\Lambda_-}{p_+^0 + p - \Sigma_+}, \\ G^A(p) &= \frac{\gamma^0\Lambda_+}{p_+^0 - p - \Sigma_-^*} + \frac{\gamma^0\Lambda_-}{p_+^0 + p - \Sigma_+^*}. \end{aligned} \tag{17}$$

The spectral function is read as

$$\rho(p) = i[G^R - G^A] = \rho_+\gamma^0\Lambda_+ + \rho_-\gamma^0\Lambda_-, \tag{18}$$

where the $\rho_\pm$ is defined by

$$\rho_\pm(p) \equiv \frac{-2\mathrm{Im}\Sigma_\mp}{(p_+^0 \mp p - \mathrm{Re}\Sigma_\mp)^2 + (\mathrm{Im}\Sigma_\mp)^2}. \tag{19}$$

Thus we get a spectral function of the Breit-Wigner type.

As for the self-energy Σ, several origins may be considered, such as the coupling to the soft mode[37] or the plasmino[38]-[41]. In this paper, however, we are interested in the physical origin of the finite viscosity so that we abandon the calculation of Σ and replace it by a following simple form,

$$\Sigma_\pm(p) = M - i\Gamma, \tag{20}$$

where M and Γ are positive constants and regarded as free parameters. Since the quasi-particles near the Fermi surface contribute main part of the integral in Eq.(15), this replacement is not so bad. The real part M is taken to be zero since it essentially shifts the chemical potential and is not sensitive to the results. The imaginary part Γ means the inverse of the mean free path (or the inverse of the life time) of the quasi-particle.

The numerical calculation was done with the temperature $T = 200$MeV and the chemical potential $\mu = 10$MeV for convenience. The momentum cutoff parameter is taken to be the usual one: $\Lambda = 631$MeV. The shear viscosity is shown in Fig.5 as a function of Γ. This figure evidently shows that the viscosity is a rapidly decreasing function of the imaginary part Γ. The viscosity seems to diverge at $\Gamma = 0$ where the quark matter becomes free gas (the mean free path is infinite). This is understood by noting that $(\Gamma/(x^2+\Gamma^2))^2 \to (\pi\delta(x))^2$ as $\Gamma \to 0$. As the Γ becomes larger, the viscosity decreases rapidly. This is also apparent because $(\Gamma/(x^2+\Gamma^2))^2 \to 0$ as $\Gamma \to \infty$. Noting that the mean free path l is Γ^{-1}, this behavior of the viscosity is consistent with the classical expression of the viscosity of gas: $\eta = \rho v l/3$ (ρ=density, v=velocity and l= mean free path of the particle).

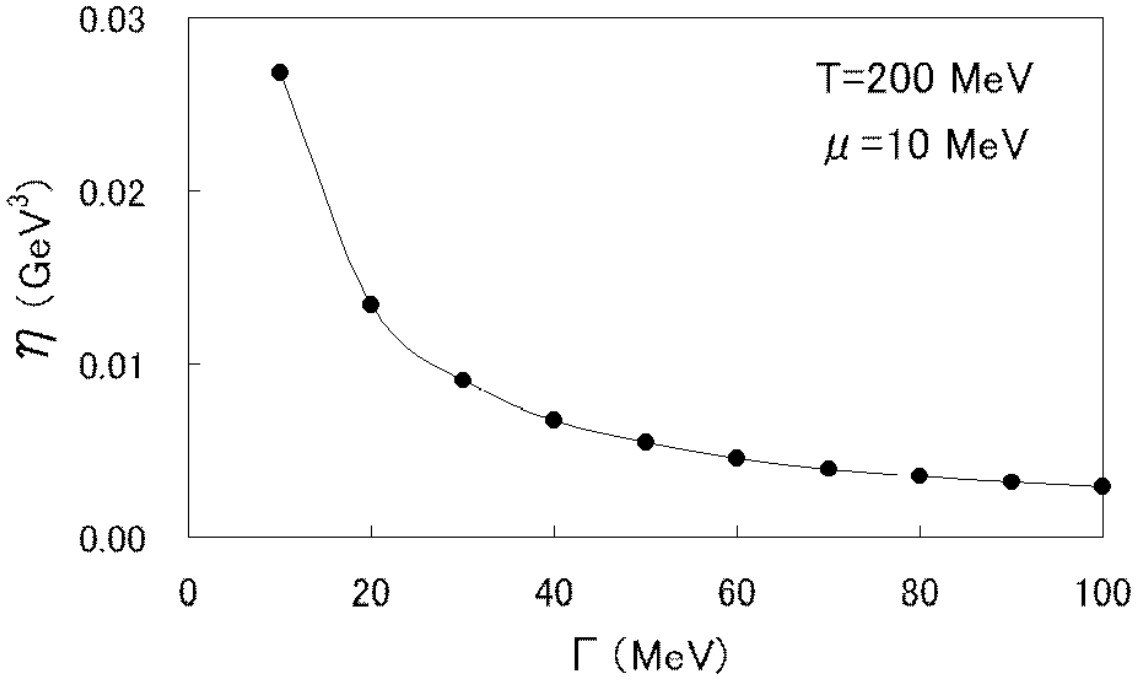

Figure 5. The shear viscosity as a function of the imaginary part of the self-energy at $T = 200$MeV and $\mu = 10$MeV

We also calculated the ratio of the shear viscosity to the entropy density in Fig.6. The entropy density is a smooth function of the temperature as compared with the shear viscosity which is divergent without the interaction between quarks. Therefore it may be approximated by

$$s = -\frac{6}{\pi^2}\int_0^\infty dpp^2[n_+\log n_+ + (1-n_+)\log(1-n_+) + n_-\log n_- + (1-n_-)\log(1-n_-)], \tag{21}$$

with the use of $n_\pm \equiv (1+\exp(p\mp\mu))^{-1}$. The ratio η/s of water in the normal state is about 30. Therefore the quark matter is a perfect fluid under strongly interacting state ($\Gamma > 1\,\mathrm{MeV}$). The lower bound predicted by the superstring theory is drawn by the dashed line in the Fig.6.

Next let us estimate the mean free path of quark and the Reynolds number. The mean free path is defined by the thermal average, $\langle(p/E)(1/2\Gamma)\rangle$. The result is shown in Fig.7 by the filled circles. Its behavior is similar to that of the shear viscosity. This means that the shear viscosity is proportional to the mean free path, which is $1 \sim 2\,\mathrm{fm}$ ($50 < \Gamma < 100\,\mathrm{MeV}$). On the other hand Reynolds number which is defined by

$$R = \frac{\text{inertia term}}{\text{viscosity term}} = \frac{l\cdot u}{\eta/(\rho_m)} \tag{22}$$

In the numerator, l and u denote typical size and velocity of the system respectively. We set to be $l = 10\,\mathrm{fm}$ and $u = 1 =$ light velocity for convenience. It is difficult to

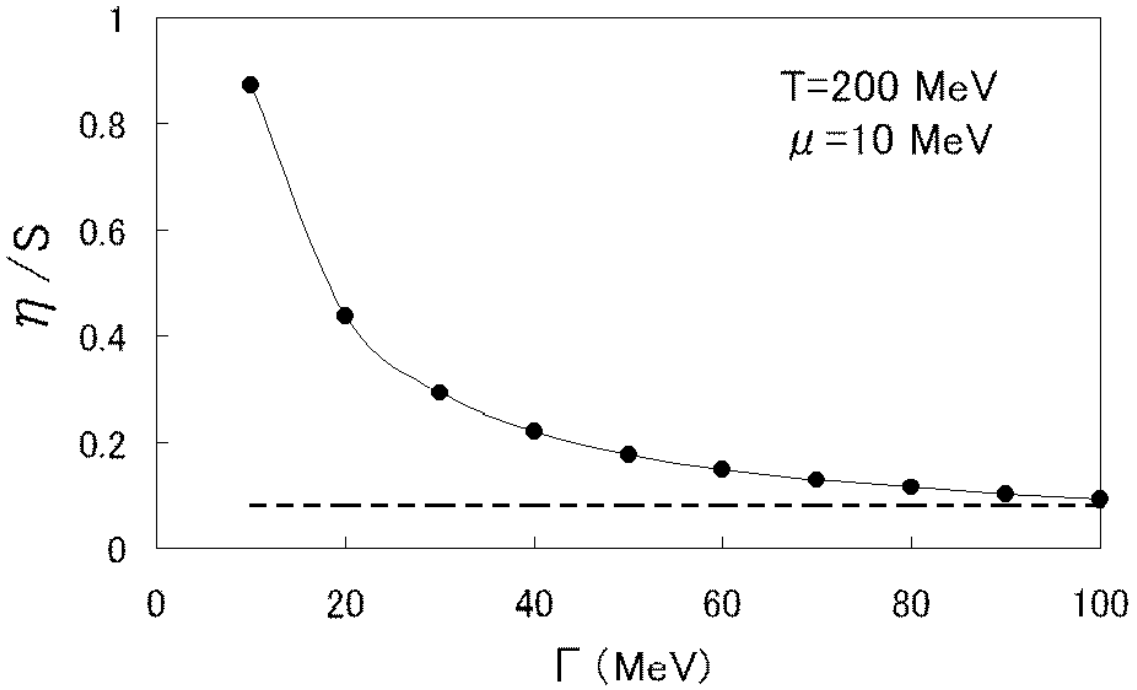

Figure 6. The ratio of the shear viscosity to the entropy as a function of the imaginary part of the self-energy at $T = 200\,\mathrm{MeV}$ and $\mu = 10\,\mathrm{MeV}$

define "mass density ρ_{m}" appeared in the denominator. When it is defined by the qusi-particle mass m times number density of the quark and antiquark, we have the result drawn by the open circles in Fig.7. Then the Reynolds number becomes $10 < R < 20$ ($50 < \Gamma < 100\,\mathrm{MeV}$) and it implies that our quark matter is close to a perfect liquid.

In conclusion, we have calculated the shear viscosity of the quark matter with the use of the Kubo formula. Using the quasi-particle RPA, we have obtained a simple formula for the shear viscosity, which is expressed by the quadratic form of the quark spectral function in the chiral symmetric phase. If we assume the Breit-Wigner type for the spectral function, the shear viscosity has been evaluated as a function of the mean free path of the quark. These results suggest that the quark matter is a nearly perfect liquid when the mean free path is shorter than 2 fm.

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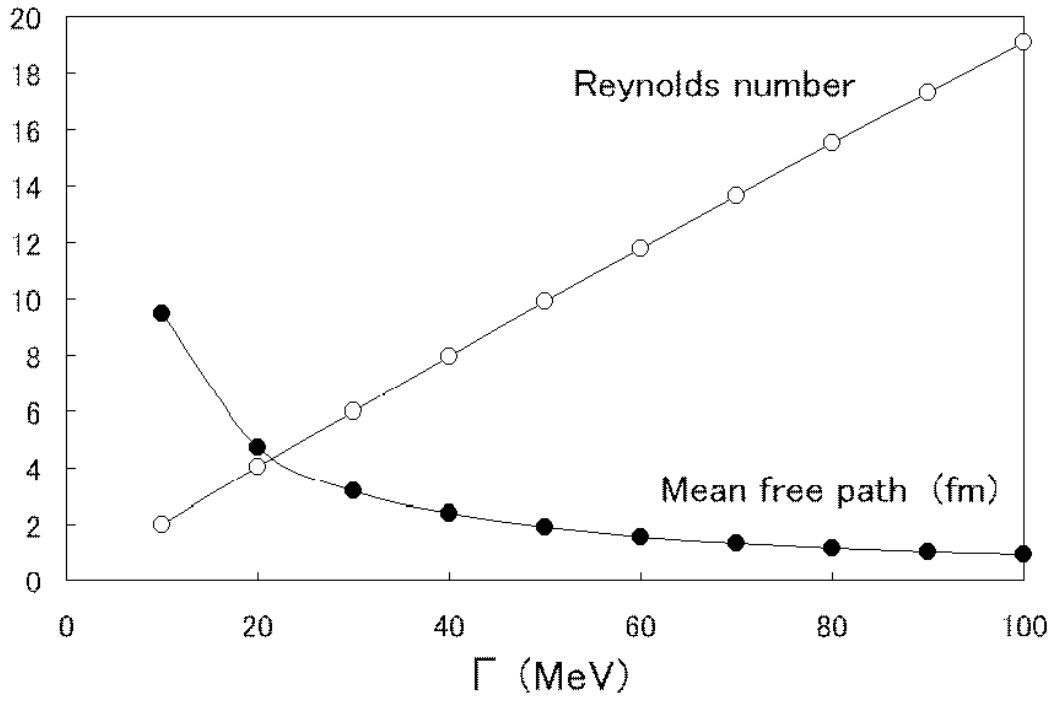

Figure 7. The mean free path of the quark (filled circles) and the Reynolds number (open circles) as a function of the imaginary part of the self-energy at $T = 200\,\mathrm{MeV}$ and $\mu = 10\,\mathrm{MeV}$

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